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RAN-2303080302030002

M.Sc. (Sem. - II) Examination September - 2025

Mathematics : Partial Differential Equations (PGMTH - 203)

Time: 3 Hours] **[Total Marks: 70**

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book

(2) Figures to the right indicate marks of the corresponding question.

(3) Follow usual notations

(4) Use of a non-programmable scientific calculator is allowed.

Seat No.:

Student's Signature

Q. 1. Answer any Two of the following: **(14)**

(a) Show that a Pfaffian differential equation in two variables always possesses an integrating factor.

(b) Solve :

(i) $\dfrac{adx}{(b-c)yz} = \dfrac{bdy}{(c-a)zx} = \dfrac{cdz}{(a-b)xy}$,

(ii) $\dfrac{dx}{mz-ny} = \dfrac{dy}{nx-lz} = \dfrac{dz}{ly-mx}$.

(c) Verify that

$yz\,(y+z)\,dx + xz\,(x+z)\,dy + xy\,(x+y)\,dz = 0$ is integrable and find its solution by Homogeneous method.

Q. 2. Answer any Two of the following: **(14)**

(a) If $f(u, v) = 0$; where u & v are function of x, y & z, f is an arbitrary function of u & v, then prove that:

$$P\frac{\partial(u,v)}{\partial(y,z)} + q\frac{\partial(u,v)}{\partial(z,x)} = \frac{\partial(u,v)}{\partial(x,y)}.$$

(b) Eliminate the arbitrary function 'f' from the following equations:

i) $f(x^2 + y^2, z - xy) = 0$,

ii) $z = f(x - y)$.

(c) Find the surface which intersect the system $z\,(x + y) = c\,(3z + 1)$ orthogonally & which passes through circle $x^2 + y^2 = 1$ & $z = 1$.

Q. 3. Answer any Two the following: **(14)**

(a) Explain Jacobi's method to solve the partial differential equation $F(x,y,z,p,q) = 0$.

(b) Show that the equations $xp - yq = 0$ & $z\,(xp + yq) = 2xy$ are compatible & solve them.

(c) Find the complete integral:

i) $p^2y\,(1 + x^2) = qx^2$,

ii) $z = p^2 - q^2$.

Q. 4. **Answer any Two of the following:** **(14)**

(a) If $\alpha_r D + \beta_r D' + \gamma_r$ is a factor of $F(D, D')$ & $\phi_r(\xi)$ is an arbitrary function of the single variable 'ξ', then prove that if $\alpha_r \neq 0$, $u_r = \exp\left(-\frac{\gamma_r}{\alpha_r}x\right)\phi_r(\beta_r x - \alpha_r y)$ is a solution of $F(D, D')\, z = 0$.

(b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} - x^2 \frac{\partial^2 z}{\partial y^2} = 0$ into canonical form.

(c) Solve the equation $q^2 r - 2pqs + p^2 t = 0$ by Monge's method.

Q. 5. **Answer any Two of the following:** **(14)**

(a) Solve the equation $(p^2 + q^2)\, y = qz$ by Charpit's method.

(b) Solve: $r + s - 2t = e^{x+y}$.

(c) Find the general integral of the linear differential equation:
$(y + zx)\, p - (x + yz)\, q = x^2 - y^2$.